QUANTITATIVE EVALUATION OF TEETH GEARS APPLYING TECHNIQUES OF VIBROPASSPORT

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Abstract. The paper presents the approach allowing quantitative evaluation of gear damages, which makes it possible to estimate the technical condition of the machine where the gear applied. The models of dynamic loads in a gear and vibrations are described, and specialties of gear transfer function are discussed. The diagnostic parameter is presented that allows quantitative evaluating the gear damage using tri-axial vibrations. The proposed algorithm of diagnostic parameter computation was experimentally tested, and the results of its validation are presented. There are conclusions about capability of the proposed algorithm for quantitative estimation of gear technical condition.

Keywords: gears vibration diagnostics, vibration path, Vibropassport.

Introduction

The basics of gear vibration modeling consider the impulse interactions between the teeth of paired gears and the vibration path to the vibration transducer. The digital signal processing technique was discussed in [1] for early damage detection by enhancing the changes which these defects produce in the signal average of vibration of the gear. The authors in [2] also considered the transfer of gear tooth meshing loads from the tooth to the shaft, rolling element bearings and the casing. Most of vibration techniques use the frequency domain, like the simplest spectral analysis [3]. An analysis of experimental data in the frequency domain is also used for diagnostic indicators search, like [4] combining the techniques of enveloping, Welch's spectral averaging and data mining-based fault classifiers. Considering the time-domain, the authors in [5] apply cyclostationary analysis and application of spectral correlation techniques on mining vibration signals. The Time Synchronized Analysis (TSA) widely used, for instance in [6], became the most usable technique for gear vibration development allowing to suppress the external vibration sources. The authors also used the above techniques in their studies for different gear types including planetary ones [7]. In [8] the study of transfer functions was discussed, including the dependence on the type of the excitation source, as well as experimental techniques of the transfer function identification were considered. Accumulated experience allows wide application of basic diagnostic methods [9] for condition monitoring of gear operation, where the task is limited to the abnormality identification.

The problem appears when the quantitative estimation of damage is required, especially aiming at predictive maintenance of complex machines. In addition to multiple vibration sources not related with the gear, the problem is caused by bearings [10] producing non-linear transformations in vibration path. This paper considers the model of gear vibrations and the technique reducing the influence of the gear vibration path. The experimentally validated technique is presented and its capability for quantitative estimation of gear damage is confirmed.

Methods and materials

The vibration model is the basis for developing the technique of quantitative damage estimation. The common model considers vibration $a(t)$ at the gear casing as a product of time-varying gear mesh load $P_d(t)$ by the transfer function $H(f)$. The typical example of dynamic load $P_d(t)$ in the gearing was described in [12] as a sequence of variable pulses (meshes). For illustration two ideal gear A and B all load impulses would be identical as it is shown in Fig. 1a. The pulses repeat with a frequency $f_g = z_a f_a = z_b f_b$, where z and f are the teeth quantity and rotation frequency of the gears. The model describes the impulse shape of a single mesh between the teeth by the normalized function $\Phi(\varphi)$, where $\varphi(t) = 2\pi f_g t$ is the gear revolution phase. The dynamic loads of the ideal gear at the operating mode m and state s of the gear can be described by the periodic function

$$P_d(t) = |\Delta P(m,s)| * \Phi(2\pi f_g t), \tag{1}$$

where $|\Delta P(m,s)|$ – modulus of variable load at the meshing frequency f_g.

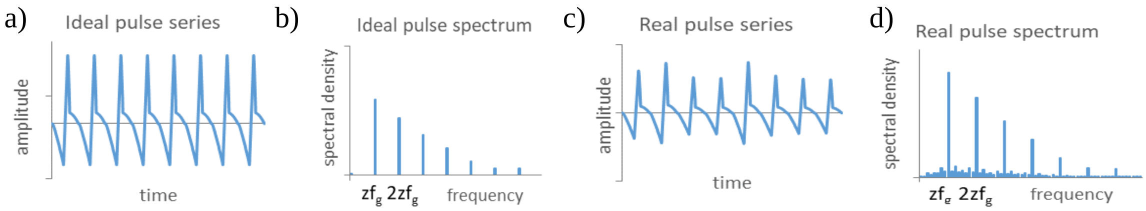

Fig. 1. **Ideal (a, b) and real (c, d) gear mesh model:**
time domain (a, c); frequency domain (b, d)

The mode of gear operation *m* depends on the torque to be transmitted and the rotation speed. The state *s* is a combination of factors, like the condition of contact surfaces, integrity of the teeth and gears. In the frequency domain, the pulse sequence is represented by a spectrum in Fig. 1b consisting of components at frequencies multiples of the gearing frequency f_g.

An actual gear has some irregularity of teeth, so the sequence of dynamic load pulses in a real gear turns out to be uneven, as shown in Fig. 1c. The shape of dynamic load pulses Φ() in the model is considered as a constant, provided by an unchanged operation mode *m* and state *s* of the gears. The function of meshing discrepancy $F[\bar{K}_a(f_a), \bar{K}_b(f_b)]$ describes the pulse amplitude variation caused by teeth irregularity of the gears A and B. The equation below models the dynamic load in the meshing of the irregular gears A and B

$$\Delta P_d(t) = |\Delta P(m,s)| * \Phi(2\pi f_g t) * F\left[\bar{K}_a(f_a), \bar{K}_b(f_b)\right]. \tag{2}$$

The tooth irregularity of each gear is characterized by an array of *z* normalized coefficients ($\bar{K}_1$, $\bar{K}_1$, …, $\bar{K}_z$). The coefficient of i^{th} tooth is the ratio of its dynamic load $P_d(i)$ to load modulus. For healthy gears the coefficient varies near 1.0 corresponding to an ideal gear, and the greater the irregularity, the more the value differs from unity. In the frequency domain, the dynamic loads of actual gearing are represented by two types of periodic spectrum components (Fig. 1d). The first, at multiples of the meshing frequency $S^P(nzf_g)$, they are called "carrier" by analogy with radio engineering. Amplitudes of these components reflect the average dynamic load on the teeth. The average shape of load pulses defines the ratio between the carrier spectral components.

The second type – sidebands $S^P(nzf_g \pm kf_a)$ and $S^P(nzf_g \pm kf_b)$ that characterize the irregularity of the A and B gear teeth. The presented simplified model of teeth meshing loads takes into account only the periodic load associated with irregularity of the gear teeth. In real gears also other factors, like the speed and gear axis fluctuation, influence the dynamic load and add the random component in the frequency domain.

For vibration diagnostics of gear damages, the model must consider both dynamic loads and the vibration path. Such model was shortly presented in [8] as a part of the Vibropassport platform. For better reflection of actual vibrations, the model considers the spatial vibration vector (measured by three-axial accelerometer) in extended frequency range. The diagnostic parameters of Vibropassport have normalized scale and common thresholds for gears of the same type that provide the capability of diagnostics based on a single observation of the operating object.

Generated by a gear the dynamic load energy is transmitted by stress waves to the transducer via rotating and stationary units. Multiple possible ways of stress wave propagation make up a vibration path as illustrated in [8]. The path of gear vibration includes fixed structural elements and bearings. Depending on the material and configuration of the fixed housing part, the length and speed of waves determine the frequency of their impact on the contact surface with the next part of the vibration path. As the structural units of a machine have different configuration, each one forms the specific frequency bands with less wave energy dissipation. Thus, the transfer characteristic of fixed structural components resembles a set of mechanic filters, whose bandwidths are determined by the modal properties of the vibration path units. Due to such passive "filtering", the fixed units of the vibration path distort the frequency structure of the transmitted signal. The gear bearings are specific elements of the gear vibration path, which transfer function [12] has two correlated parts. The bearing kinematics determines a deterministic part, while the raceways-roller interactions cause a random one. In contrast to fixed units, the bearing not only distorts the frequency composition of stress waves but also transposes part of the transferred energy to other frequency ranges. The reason of energy transposition in a frequency domain is dynamic resonances between load meshing components and bearing interactions. The complicated structure of the transfer function may be illustrated thanks to the correlation between its deterministic and random parts. As an example of the gear transfer function the spectrum of random vibration measured on the casing of the gearbox is presented in Fig. 2a.

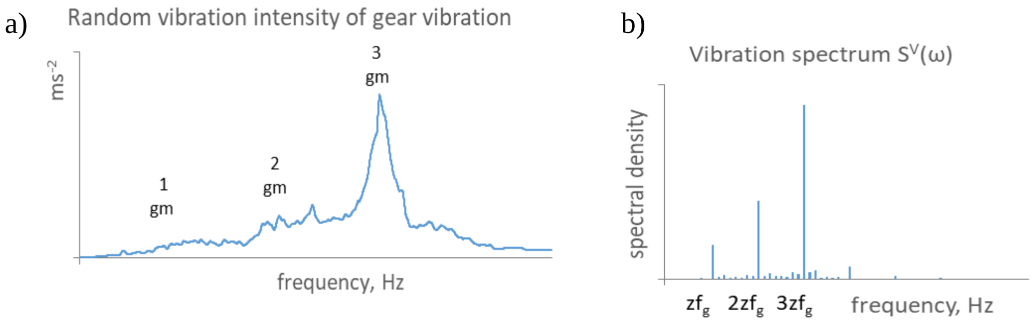

Fig. 2. **Vibration of the gear: random component (a), vibration spectrum (b)**

The peak of the random vibration component reflects the matching between the third harmonic of the gear mesh frequency and the multiple rolling frequency harmonic of the bearing. The dynamic loads with the spectrum as in Fig. 1d passing through the vibration path with the spectrum like on Fig. 2a transform into vibration spectrum like in Fig. 2b. Thus, the transfer function of the gear vibration path $H(f)$ significantly distorts the frequency structure of meshing dynamic loads. The transfer function of the gear could be considered stationary until the conditions of the vibration path remain unchanged. Variable operation mode may modify the transfer function if the temperature or bearing lubrication change. An accelerometer typically used for vibration measurement is also the part of vibration path, and each measurement direction of vibration differently transmits the dynamic load of the gear. Therefore, the spatial response provided by the tri-axial accelerometer requires consideration of the transfer function in each measurement direction separately.

With the stationary transfer function, vibrations reflect the dynamic load of meshing gears with essential distortion. Following to equation (1) the gear vibration model is expressed as the product of the dynamic loads (2) and the transfer function

$$a_{x,y,z}(t) = H_{x,y,z}(f) * |\Delta P(m,s)| * \Phi(2\pi f_g t) * F\left[\bar{K}_a(f_a), \bar{K}_b(f_b)\right]. \tag{3}$$

As it can be seen from the equation, the vibration pattern of the gearing is also represented by a pulse sequence with a base frequency f_g. The fluctuation of the vibration pulse amplitude in time also corresponds to irregularity of both gears $F[\bar{K}_a(f_a), \bar{K}_b(f_b)]$. However, the amplitude and shape of pulses $H(f)*|\Delta P(m,s)|*\ \Phi(2\pi f_g t)$ change significantly compared to the dynamic loads due to the non-linear transfer function. In the frequency domain, similar to dynamic loads, two types of vibration components reflect the vibration model. The components with frequencies $S^V(nzf_g \pm kf_a)$ and $S^P(nzf_g \pm kf_b)$ characterize the irregularity of both gears, which in the model is described in the normalized function $F[\bar{K}_a(f_a), \bar{K}_b(f_b)]$. The components $S^V(kzf)$ correspond to vibration response $H_{x,y,z}(f)$ on average amplitude and shape of dynamic loads $|\Delta P(m,s)|*\ \Phi(2\pi f_g t)$. Fig. 2b illustrates the ratio between the spectral components in comparison to the spectrum in Fig. 1d.

The model of actual vibrations measured on a casing in addition to the gear considers also external vibration sources and the multiplying factor

$$S^G_{x,y,z}(f) = K^M_{x,y,z}\left\{H_{x,y,z}(f) * \left[|\Delta P(m,s)| * S^V\left(nzf_g \pm k_a f_a \pm k_b f_b\right)\right] + \sum S^{ext}(f)\right\} + \sum S^{EI}(f). \tag{4}$$

External vibrations not related to the gear may be presented as the sum of vibration components $\Sigma S^V(f)$. There is also a factor $K^M_{x,y,z}$ that may multiply the signal of the transducer, for instance, the dependence of its sensitivity on the temperature or the measurement channel gain. The sum of electrical interference and signal noising factors $\Sigma S^{EI}(f)$ is presented also as an additive part. Thus, the deterministic vibration model takes into account both additive and multiplicative components of the measured vibration signal.

Based on the vibration model (4) the teeth diagnostic parameter (TDP) of Vibropassport estimates irregularity of the gear. TDP of the pinion A estimates the teeth irregularity using the ratio between the carriers $S^V(nzf_g)$ in equation (4) characterizing the averaged load pulses and the sidebands $S^V(nzf_g \pm kf_a)$ proportional to these pulses irregularity. When local limited damage in the gear A appears, the sidebands $S^V(nzf_g \pm kf_a)$ change, while the carriers $S^V(nzf_g)$ are considered unchanged. The computation algorithm of TDP includes four stages: TSA of the vibration signal, spectral analysis of enhanced waveform, spectral components identification, and TDP computation. TSA considers splitting the signal record into sections corresponding to the gear revolution periods (waveforms) and these waveforms averaging. The enhanced vibration waveforms for three measurement directions are the result of TSA procedures. Then, spectral transformation of enhanced waveforms provides three order spectra, in which all components correspond to the gear only. The above procedures allow suppressing vibrations of extraneous sources, like components $\Sigma S^{ext}(f)$, and $\Sigma S^{EI}(f)$ improving the signal-to-noise ratio. TDP for pinion A with z_a teeth from the three-axial accelerometer is computed using equation (5)

$$K_T^a = \frac{1}{3}\sum_{x,y,z}\left\{\sqrt{\sum_{k=1}^{K}\left[\sum_{n=1}^{N} S_{x,y,z}^{G}\left(nzf_g \pm kf_d\right)/\,S_{x,y,z}^{G}\left(kzf_g\right)\right]^2}\right\}, \tag{5}$$

where $N = int(z_a/2)$ – number of sideband components and K is the number of carriers in the defined frequency range.

The way of K^a_T computation provides two benefits. First, it excludes the impact of the multiplying factor K_M in equation (4), by using the ratio of sidebands to the carrier. Second, the influence of the transfer function is suppressed. Due to the influence of the transfer function, the estimates of the sidebands/carrier ratio are under- or overestimated in different frequency ranges. Averaging the estimation errors with different signs over multiple ranges the algorithm reduces the total error.

Three basic types of tooth damage are considered: damage of the contact surface simulating pitting (Fig. 3a), fatigue crack at the tooth's root (Fig. 3c) and a broken tooth.

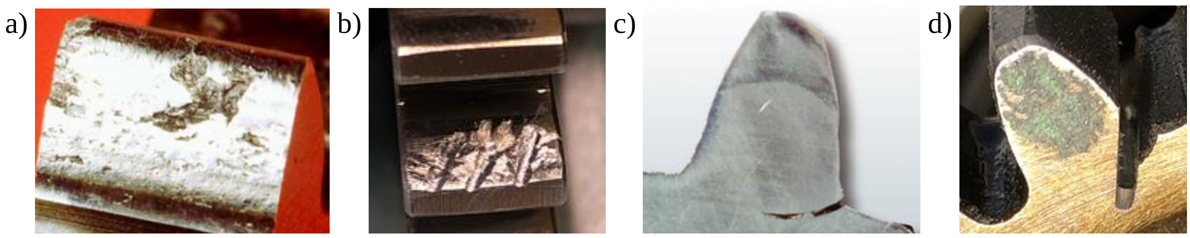

Fig. 3. **Actual (a, c) and simulated (b, d) tooth damages**

The vibration model describes damages using coefficients of teeth irregularity. For instance, the damage of the i^{th} tooth surface of the gear A is modeled by modifying the corresponding coefficient $\bar{K}_i(f_a)$. The profile of any specific irregularity of the gear was simulated by combination of interacting gear coefficients. Modeling the damages, like in Fig. 3b, the intensity of sidebands increased, while the carriers associated with the shape of the tooth interaction pulse remained almost unchanged. The entire tooth breakage was modeled by zeroing out the coefficient of the relevant tooth. The vibration model of the broken tooth demonstrated different change: sidebands increase, while total energy of carriers decreases due to "blurring" of the pulse. This change led to growth of the sidebands/carrier ratio, and that means the equation (5) is usable for the case of a broken tooth also, although the assumption of the unchanged pulse shape is not met.

The capabilities of TDP were experimentally validated using seeded faults of different type and scale. The experimental study was carried out using the test rig based on Ai-24 aircraft gas turbine engine. The pinion of the AC generator drive gear was the object for seeded fault implementation. The external electric motor was driving the engine's rotor through the gearbox (inverse mode), providing the pinion dynamic loading. The data measurement system recorded the signals of the rotation speed and of the 3-axial accelerometer mounted on the engine casing. For each seeded fault the measured signal were processed with TSA procedures and then TDP was calculated using formula (5). Table 1 indicates the data of 10 technical states of the tested pinion, for which TDP were computed.

Initial damage test (indexed "s0.3") was made with a slight damage to the contact surface (depth 0.1 mm, damaged 30% of the contact surface) that simulated initial pitting of the tooth. As a typical defect the rougher damage illustrated on Fig. 3b was used (depth 0.1-0.3 mm, damaged 100% of the contact surface). Such fault was seeded step by step to the pinion's teeth, and the signals of five tests were recorded with 1, 3, 4, 13, and all 26 damaged teeth (indexed with letter "s").

Table 1

Teeth diagnostic parameter (TDP) for 10 pinion states

State	**Health**	**Teeth surface damaged**						**Cut & undercut teeth**		
Index	H	s0.3	s1	s3	s4	s13	s26	1c	1c/14uc-	1c/14uc +
Damaged teeth	-	0.3	1	3	4	13	26	-	-	-
TDP	1.47	1.78	1.81	1.83	2.12	2.14	2.74	4.30	11.3	15.6
TDP change, %	-	21	23	25	45	46	87	193	669	964

More critical faults were arranged by cutting off the pinion's teeth. First, the tooth 1 was cut off ("1c"). As the pinion interacts with two opposed idler gears, for the next tests the opposite tooth 14 was undercut in two steps: smaller ("1c/14uc-") illustrated in Fig. 3d, and then deeper ("1c/14uc + "). Fig. 4 illustrates the dependence of TDP on the scale and type of seeded faults.

a) b)

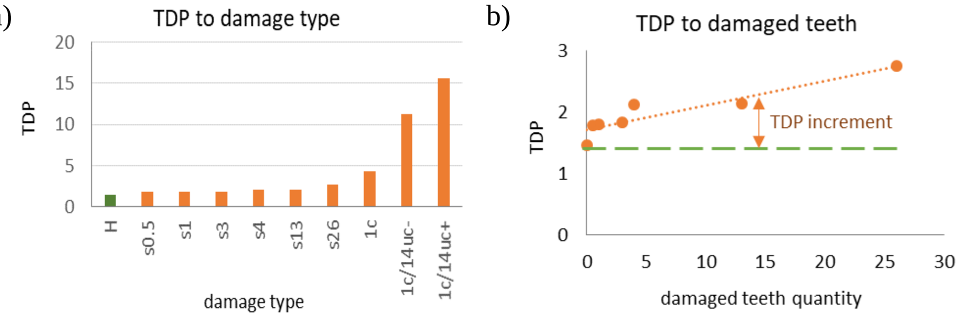

Fig. 4. **Diagnostic parameter dependence on damaging (a), damaged teeth quantity (b)**

Discussion and conclusions

Table 1 presents TDP values for each type of seeded faults and change of the parameter in comparison to healthy (reference) state. As it can be seen, the value of the diagnostic parameter increases as the gear teeth damage extends. Although there is no common scale for all types of damage, the relationship between the diagnostic parameter and increasing dynamic loads in the gearing is evident (Fig. 4a). For example, removing the tooth 1 in addition to all damaged teeth led to an increase of TDP from 2.74 to 4.30 (Table 1). Adding more damage into the tooth 14 that opposes to the 1st one and simultaneously engages another idler gear, the dynamic loads and the diagnostic parameter increase. The number of damaged teeth is the real scale factor of the specific damage type and Fig. 4b demonstrates TDP dependence on this factor. Practically linear dependence allows quantitative estimation of gear damage using the teeth diagnostic parameter.

The proposed model of gear vibration made it possible to formulate the technique and the diagnostic parameter capable of providing a quantitative assessment of gear teeth damage. Such ability became possible thanks to reducing the influence of the transfer function of the gear vibration path mainly influenced by the bearings. This is achieved by using an extended frequency range and measuring the full vibration vector with a 3-axis accelerometer simultaneously with the rotation speed. The calculation method uses a TSA algorithm, which requires the use of vibration and speed signals. The relationship between the diagnostic parameter and the damage scale verified experimentally, confirmed both the adequacy of the model used and the algorithm applied for its calculation. The possibility of quantitative assessment of gear damage using the diagnostic parameter of teeth has been confirmed.

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Author contributions

Conceptualization, methodology, validation, formal analysis, A.M.; software, investigation, data curation, visualization, P.D; writing – original draft preparation, A.M.; writing – review and editing, P.D. All authors have read and agreed to the published version of the manuscript.

References

[1] McFadden P., Smith J. A Signal Processing Technique for Detecting Local Defects in a Gear from the Signal Average of the Vibration. Proceedings of The Institution of Mechanical Engineers Part C-journal - 199. 1985. Pp. 287-292. DOI: 10.1243/PIME_PROC_1985_199_125_02.

[2] McFadden P., Smith J. Effect of Transmission Path on Measured Gear Vibration. Journal of Vibration Acoustics Stress and Reliability in Design. 1986. 108. 377. DOI: 10.1115/1.3269353.

[3] Lahdelma S., Juuso E., Immonen J. Advanced condition monitoring of epicyclic gearboxes. In: 10th International Conference on Condition Monitoring and Machinery Failure Prevention Technologies, CM 2013/MFPT Poland, 2013.

[4] Zhuang L. Experimental Analyses of Vibration and Noise of Faulted Planetary Gearbox. In: Proceeding of Inter-noise 2014, Melbourne, Australia.

[5] Bartelmus W., Zimroz R. Vibration diagnostic method for planetary gearboxes under varying external load with regard to cyclostationary analysis. In: 527/2009 Wrocław, 2009.

[6] Bechhoefer E., Kingsley M. "A Review of Time Synchronous Average Algorithms," in Proceedings of the Annual Conference of the PHM Society 2009, Vol. 1 No. 1, 2009.

[7] Mironov A., Mironovs D. Condition Monitoring of Helicopter Main Gearbox Planetary Stage: Selected Papers from the 18th International Conference on Reliability and Statistics in Transportation and Communication, RelStat'18, 17-20 October 2018, Riga, Latvia.

[8] Mironov A., Doronkin P., Mironovs D. Research and Practical Application of Vibration Transfer Functions in Diagnostics of Jet Engine: Selected Papers from the 18th International Conference on Reliability and Statistics in Transportation and Communication, RelStat'18, 17-20 October 2018, Riga, Latvia. DOI: 10.1007/978-3-030-12450-2_40.

[9] Aherwar A., Khalid S. Vibration analysis techniques for gearbox diagnostic: A review. Int. J. Adv. Eng. Technol. 3., 2012 pp. 4-12.

[10] Sun X., Wang T., Zhang R., Gu F., Ball A.D. Numerical Modelling of Vibration Responses of Helical Gears under Progressive Tooth Wear for Condition Monitoring. Mathematics 2021, 9, 213. DOI: 10.3390/math9030213

[11] Krantz T.L. Gear Tooth Stress Measurements of Two Helicopter Planetary Stages. In: 6th International Power Transmission and Gearing Conference sponsored by the American Society of Mechanical Engineers. NASA Technical Memorandum 105651. 1992. [online][11.02.2023] Available at: https://ntrs.nasa.gov/api/citations/19920017312/downloads/19920017312.pdf

[12] Mironov A., Doronkin P., Priklonsky A. Adaptive Vibration Diagnostic Technique for Bearings Condition Monitoring of Complicated Machines. In book: Advances in Condition Monitoring of Machinery in Non-Stationary Operations. 2016. pp. 425-438. DOI: 10.1007/978-3-319-20463-5_32.